MATHEMATICS ENRICHMENT CLUB.
Solution Sheet 5, June 3, 2014 [1]

1. To make up 100 we can assume that all the numbers we sum are two digit, if even one was three digit there would be no way to sum to 100. Let's say we write the numbers as $10a_i + b_i$, where the a_i and b_i follow the condition that all digits 0-9 be used exactly once. Now let the sum of the a_i be A and the sum of the b_i be B. The value of the sum is then

$$10A + B = n$$

but we also know that

$$A + B = 45$$

just by adding all the numbers from 0 to 9. By subtracting the second from the first we see that

9A 6 Ted [(9)]TJ/F19 11.9552 Tiio67d [(n)m+

3. Draw the base of the triangle as AB. The base, AB, is made up of 3 segments x, y and z. The middle, y, is an edge of one of the inner triangles while the outer two are equal length to sides of the two other inner triangles, as they are sides of parallelograms which are each opposite an inner triangle. All inner triangles are similar to each other, and since their areas are in ratio 12 : 27 : 75 or rather 4 : 9 : 25 then their sides are in ratio 2 : 3 : 5. Inlcuding the larger triangle in these sides ratios all 4 triangles have their sides in ratio 2 : 3 : 5 : 10, and so their areas are 4 : 9 : 25 : 100. So if the inner triangles have area 12,27 and 75 the larger triangle has area 300.

4. First note that we can buy exactly 10 = 5 + 5 donuts. Now if gure out the 9 ways

where we have used $a_{n+1} = a_n + a_{n-1}$ in the first line and rearranged $a_n = a_{n-1} + a_{n-2}$ to get $a_n - a_{n-1} = a_{n-2}$

Senior Questions

1. (a) Using the rule for addition

$$[A+B]_{ij} = [A]_{ij} + [B]_{ij} = [B]_{ij} + [A]_{ij} = [B+A]_{ij}:$$

(b) Similarly

$$[A+(B+C)]_{ij} = [A]_{ij} + [B+C]_{ij} = [A]_{ij} + [B]_{ij} + [C]_{ij} = [A+B]_{ij} + [C]_{ij} = [(A+B)+C]_{ij}:$$

2. (a) Using the rule for matrix multiplication

$$\begin{aligned}[A(BC)]_{ij} &= \sum_{k=1}^{n} [A]_{ik}[BC]_{kj} \\ &= \sum_{k=1}^{n}\sum_{l=1}^{n} [A]_{ik}[B]_{kl}[C]_{lj} \\ &= \sum_{l=1}^{n} [AB]_{il}[C]_{lj} \\ &= [(AB)C]_{ij}\end{aligned}$$

(b) Similarly

$$[AB]_{ij} = \sum_{k=1}^{n} [A]_{ik}[B]_{kj}$$

and

$$[BA]_{ij} = \sum_{k=1}^{n} [B]_{ik}[A]_{kj}$$

but since $[B]_{ik}$ does not necessarily equal $[B]_{kj}$ the two above are not equal. Take for example

$$\begin{matrix}1 & 1\\ 0 & 0\end{matrix}\quad\begin{matrix}1 & 0\\ 1 & 0\end{matrix} = \begin{matrix}1 & 1+1 & 1 & 1 & 0+1 & 0\\ 0 & 1+0 & 1 & 0 & 0+0 & 0\end{matrix} = \begin{matrix}2 & 0\\ 0 & 0\end{matrix}$$

while

$$\begin{matrix}1 & 0\\ 1 & 0\end{matrix}\quad\begin{matrix}1 & 1\\ 0 & 0\end{matrix} = \begin{matrix}1 & 1+0 & 0 & 1 & 1+0 & 0\\ 1 & 1+0 & 0 & 1 & 1+0 & 0\end{matrix} = \begin{matrix}1 & 1\\ 1 & 1\end{matrix}:$$

3. Looking at the commutation equation for mulitplication above, let's let $B = I$. We need

$$\sum_{k=1}^{n} [A]_{ik}[I]_{kj} = A_{ij}$$

and

$$\sum_{k=1}^{n} [I]_{ik}[A]_{kj} = A_{ij}.$$

If we have every entry of I equal to zero, except $[I]_{ii} = 1$ for $i = 1, \ldots, n$ we can see that the above two are satisfied, so

$$[I]_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases};$$

or

$$I = \begin{pmatrix} 1 & 0 & \cdots & & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & & 0 \\ 0 & \cdots & & 0 & 1 \end{pmatrix}$$

works. Is this the only one? Suppose there is another one I_2 which works as well, then

$$I I_2$$

.