

MATHEMATICS ENRICHMENT CLUB.

Problem Sheet 12, August 2, 2015 ¹

- (a) Show that the number $2017^6 - 46$ is not prime.
(b) Show that the number $2017^6 + 46$ is not prime.
- The polynomial $x^3 + px^2 + qx + r$ as a factor of $(x - 5)$ and a remainder of 24 when divided by $(x - 1)$. Find the values of p and q .
- Let $A(-13; 14)$; $B(-3; -5)$ and $C(7; 11)$ be the coordinate of vertices of a triangle $\triangle ABC$. Find the coordinate of the point P inside $\triangle ABC$ such that the triangle $\triangle PBC$ is equilateral.
- An arithmetic sequence has positive integral entries. The sum of some 4 consecutive terms is 30. The sum of some 5 consecutive terms is 30. Find the least number of terms overlapping in the two subsequences.
- Let $f(x) = \frac{4^x}{4^x + 2}$. Calculate

$$f\left(\frac{1}{2015}\right) + f\left(\frac{2}{2015}\right) + \dots + f\left(\frac{2014}{2015}\right) :$$

- In a convex quadrilateral $ABCD$ the diagonals are perpendicular. Points M and N are marked on the sides AD and CD respectively. Suppose the $\angle ABN$ and $\angle CBM$ are right-angles, prove that the lines AC and MN are parallel.

¹Some problems from UNSW's publication *Parabola* and the Tournament of Towns in Toronto.

Senior Questions

1. Let $[x]$ denote the greatest integer less than or equal to x . If α and β are the roots of the function $f(x) = x^3 + x^2 - 5x - 1$, find $[\alpha] + [\beta] + [\alpha\beta]$.
2. Show that if n and m are positive integers, then $(n!)^m = (m!)^n (n!)^m$.
3. Find all positive numbers x and y such that

$$x^{x+y} = y^{x+2y} \quad \text{and} \quad x^{2x+y} = y^{x+4y}:$$