

MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 12, August 8, 2016

1. Find the smallest possible integer n , such that $n + 2n + 3n + \dots + 99n$ is a perfect square.

2. Let

$$f(n) = \frac{1 + 2 + 3 + \dots + n}{n};$$

Evaluate $f(1) + f(2) + f(3) + \dots + f(99) + f(100)$.

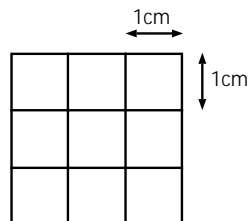
3. P is a point inside a convex polygon whose sides are all equal in length. Perpendiculars are constructed from P to the sides of the polygon. Show that the sum of the lengths of the perpendiculars is the same for all positions of P .

4. Let A , B and C be integers. Find the smallest possible prime p , such that

$$\frac{x^2 - p}{(x - 2)(x - 3)(x - 5)} = \frac{A}{x - 2} + \frac{B}{x - 3} + \frac{C}{x - 5};$$

5. Is it possible to make a 4×4 square lattice of size 4 cm by 4 cm by using

- (a) 5 pieces of thread, each 8 cm long?
- (b) 8 pieces of thread, each 5 cm long?



6. Find the last two digits of $\sqrt{4^{2016} + 2 \times 6^{2016} + 9^{2016}}$.

Senior Questions

1. Given 2 three digit numbers a and b and a four digit number c . If the sum of the digits of the number $a + b$, $b + c$ and $c + a$ are all equal to 3, find the largest possible sum of the digits of the number $a + b + c$.
2. Are there integers $a; b$ which satisfy

$$5a^2 - 7b^2 = 9?$$

Either find them or show that they do not exist.

3. Prove that there is no convex eight sided polygon with all angles equal and the sides distinct integers.