

MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 6, June 11, 2018

1. Solve x

Senior Questions

1. Prove that the square of the n th triangular number is the sum of the first n cubes, i.e.

$$\sum_{k=1}^n k^2 = \sum_{k=1}^n k^3$$

2. Find the limit $\lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3}$.

3. Let $f(x) = xe^x$.

(a) Draw the graph of $y = f(x)$, clearly indicating any stationary points on your diagram.

(b) For $x > e^{-1}$, $f(x)$ has an inverse, $f^{-1}(x)$. Add the graph of $y = f^{-1}(x)$ to your diagram.

(c) This inverse is the principal branch of the Lambert W function, and is also known as the Omega function or the product-log function. We will denote it by $W(x)$. Show that

$$\frac{dW}{dx} = \frac{W(x)}{x(1 + W(x))}$$