with it (1, 2, 3 and 4), so $\varphi(5) = 4$. In contrast, there are only two integers less than 6 that are coprime with it (1 and 5), so $\varphi(6) = 2$.

(a) Find $\varphi(12)$ and $\varphi(30)$.

(b) Suppose that p is prime. Find $\varphi(p)$, $\varphi(p^2)$ and $\varphi(p^3)$.

(c) If p and q are two distinct primes, find $\varphi(pq)$.

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is a triangle with E and F being the
Prove that EF is parallel to BC and half the

Senior Questions

1. (a) Show that $n^4 - 6n^3 - 18n^2 + 6n + 1 = (n^2 - 3n - 1)^2 - 25n^2$,

   (b) Hence find all integers n such that $n^4 - 6n^3 - 18n^2 + 6n + 1$ is prime.

2. How many real roots does the equation $x = 3(1 - \sin x)$ have? Use Newton's method to find an approximate value of the smallest one and hence find the largest one.

3. Let ABC be a triangle. The median of a triangle is the line segment that connects the midpoint of one side to the opposite vertex. Prove that the medians of ABC intersect at a single point, called the centroid, and that the centroid divides the median in the ratio $1 : 2$, with the centroid lying twice as far from the vertex as from the foot of the median.