MATHEMATICS ENRICHMENT CLUB.

Solution Sheet 6, June 18, 2018

1. Firstly, we note that $2x + 5y \neq 0$. Then

$$x + 3y$$

x	y	z	$x+y$	$x+z$	$y+z$
1	1	1	2	2	2
1	1	3	2	0	0
1	3	3	0	0	2
3	3	3	2	2	2

We know from part (a) that squares are either 0 or 1 mod 4, so none of these combinations works.

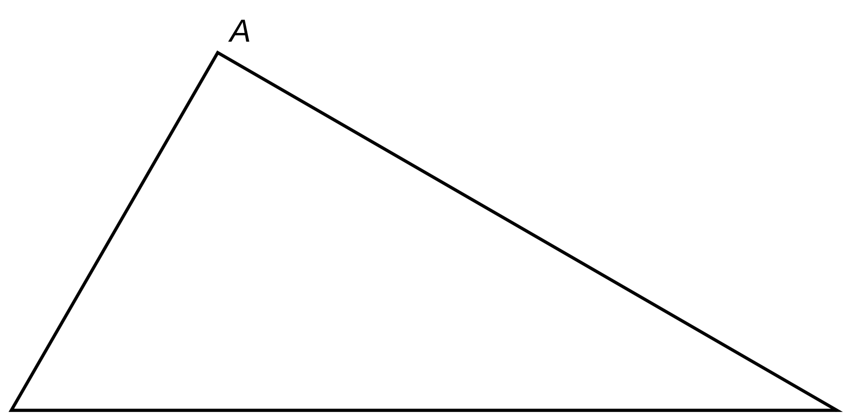
Without loss of generality, suppose that x is even but both y and z are odd. Then the table is as follows:

x	y	z	$x+y$	$x+z$	$y+z$
0	1	1	1	1	2
0	1	3	1	3	0
0	3	3	3	3	2
2	1	1	3	3	2
2	1	3	3	1	0
2	3	3	1	1	2

Once again, we see that this does not work if two of the integers are odd. Consequently, at most one of the integers is odd. (It can be seen that $x, y \equiv 0$ and $z \equiv 1$ or $x, y \equiv 2$ and $z \equiv 3$ mod 4, for instance, might work.)

(c) Try 19, 30 and 6.

5. Let KG, LG and GM be perpendiculars from G to AB, AC and BD, respectively. Then $\triangle BKG$ and $\triangle BGM$ are two right triangles with a smaller angle and a hypotenuse in common, so $\triangle BKG \cong \triangle BGM$. Thus $GK = GM$. By a similar argument, it can be shown that $GM = GL$. Consequently, $\triangle GAK \cong \triangle GAL$, and so GA bisects $\angle BAC$, as required.



Senior Questions

1. Use mathematical induction.

2. Recall that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(2n+1)(n+1)}{6}$. Then $\lim_{n \to \infty}$ $1^2 + 2^2 + 3^2 +$

But $e^y = \frac{x}{y}$, and so

$$\begin{aligned} 1 &= \frac{x(1+y)}{y} \quad \frac{dy}{dx} \\ \frac{dy}{dx} &= \frac{y}{x(1+y)} \end{aligned}$$

And since $y = W(x)$, the result follows.