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01 1 238! +7<Z8 1 !>3 \ !)>3^89a! ! .L! NZMO! `TRNXSTc! aPX! dM``! `TYSL! RYLRTSO! PQ! NZT! M \ \XLT!0a0NT \ !YLU!NZT!SP`T!PQ!NZT!M \ \XLT!0a0NT \ !ML!RPLNSP``ML [!RYLRTS,!!

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 $+ PX!NZTL!TfY \ MLT!NZT!ONSXRNXST!PQ!OP \ T!`a \]ZPMU!PS[YLOc!dZMRZ!dM``!ZT`]!aPX!NP! \\ _TNNTS!XLUTSONYLU!NZT!YLYNP \ a!PQ!NZT!M \ \ XLT!OaONT \ ,!!$

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- ", %OML[!NZT!PM'!M \ \ TSOMPL!`TLO!3?@@f5c!TfY \ MLT!Y!LX \ _TS!PQ! dZMNT!RT``O,!) YeT!Y!USY dML[!PQ!TYRZ!UMQQTSTLN! \ PS]ZP`P[MRY`!Na]T!PQ! dZMNT!RT``!NZYN! aPX!OTT!YLU!MUTLNMQa!NZT \ c!_a!STQTSTLRT!NP!NZT!M \ Y[TO!Y_P^T,!

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- >.!!! *!] YMTLN! dZP! LTTUO!] YOOM^T! N \ \ XLMgYMPL! QPS! OLYET_MT!_XN! NZT! PL`a! $Y^{Y}Y_{T}'T_{V}N_PUa!RP \setminus TO!(SP \setminus !ZPSOTO, !! 2ZT!(P^{P}d)ML[!N \setminus]PSNYLN!YO]TRN!$ \X0N!_T!NYeTL!MLNP!YRRPXLN!_TQPST!YU \ MLMONTSML [!NZT!ZPSOT!YLNM_PUaV!
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 $YRW^{YNT!}PL[!NTS \ !]SPNTRWPL!_a!NZT!OW \ X`YMPL!P0! \ T \ PSa!: !$ RT``0,! ! ! =,!!!!!!! 'TSX \ !OMReLTOO!MO!Y!RPLUMMPL!dZTL! \ YOOM^T!Y \ PXLNO!PO!YLM[TL!RP \ MLML[!

OZPSN!NTS \ !] SPNTRNNPLc!YLU!M \ \ XLMgYNNPL! d MNZ!NP f PMU! d M`!

3 5! OT^TL!aTYSO!Y [Pc!NZTST!MO!LP!LTTU!QPS!NSTYN \ TLN! 3R5!!! * !_PPONTS!0ZPN!PQ!NPfPMU!dM`!QYN`!NP!]SP^MUT!]SPNTRNAT!N \ \ XLMA!

aTYSO, !6Y00M^T!M \ \ XLMgYNMPL!MO!NZTST0PST!LTRT00YSa!

* 0!NZT! d PXLU!M0!ST`YMM^T`a! \ MLPS!YLU!R`TYLc!YLU!NZT!]YMMTLN!M0!YL! YUX`N! d NNZ!Y!RP \]`TNT!^YRRNLYNNPL!ZNONPSac!YLU!Y! PPONTS!OZPN!

TRYX0T!P0!NZT!`P00!P0! \ T \ PSa!RT``0!Y0NTS!NZT!]Y00Y[T!P0!0T^TL!

*! PPONTS!P0!0ZPN!P0!NPfPMU!d M^`!0NM \ X`YNT!`PL[G`M^TU! \ T \ PSa! RT``Oc! d ZMRZ! d M``! [TLTSYNT!Y!] SPNTRM^T!YLNM_PUa!STO] PLOT,!

 $6Y00M^T!N \setminus XLMgYNMPL!dMXZ!NTNYLX0!N \setminus XLP[`P_X`NL!dM`!]SP^MUT!$

- NP!NZT!UT^T`P] \ TLN!PQ!]SPNTRM^T!YLM_PUMTO,!
- 3Y5!!! (f]POXST!NP!_YRNTSWY`!NPfNL!dN``!YRM^YNT! \T \PSa!RT``Oc!`TYUNL[!

*!"-GaTYSGP`U!]YNMTLN!ZY0!Y!R`TYL!dPXLU!QSP \ !_SPeTL![`YOOc!_XN!OZT! STRTM^TU!Y!QX``!RPXSOT!PQ!NZT!NTNYLXO!^YRRMLT!YO!Y!RZM`Uc!YLU!Y! PPONTS! ^YRRMLYMPL!OT^TL!aTYSO!TYS`MTS, ! H ZMRZ!PQ!NZT!QP``PdML[!ONYNT \ TLNO!MO!

RPSSTRN!YLU!]SP^MUTO!NZT!_TON!NSTYN \ TLN!ONSYNT[as!

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?, .N!MO!eLPdL!NZYN!]YMTLNO!dMNZ!UMY_TNTO!YST! \ PST!OXORT]NM_`T!NP! UT^T`P]ML[!MLQTRNMPLO,!6TSQPS \ !YL!MLNTSLTN!OTYSRZ!NP!QMLU!PXN!ZPd! UNY_TNTO! \ Ya!YQQTRN!NZT!M \ \ XLT!QXLRNMPLO,!!
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- 2P!YRRXSYNT`a!UTNTS \ MLT!NZT!RPLRTLNSYNMPL!PQ!RT``0!ML!Y![M^TL!0X0]TL0MPL!
- 2P!YRRXSYNT`a!UTNTS \ NLT!NZT!^NY_N`Na!PQ!Y! [N^TL!OX0] TLONPL!PQ!RT``0,!!
- 2P!_T!Y_`T!NP!STYUM`a!]TSQPS \ !RY`RX`YNNPLO!NL^P`^NL[!RT``!RPXLNOc!^MY_M`NNa! YLU!RPLRTLNSYMPLO.!

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8, *0!aPX!eLPd!NZT!^P`X \ T!NZYN!0M0!Y_P^T!TYRZ!0WXYSTc!aPX!RYL!RPXLN!RT``0! YLU! NZTL! RY`RX`YNT! NZT! RPLRTLNSYMPL! PQ! NZT! RT``O! NL! NZT! YUUTU! OP`XNMPL,! 7 Pd! \ YLa!OWXYSTO! aPX! RPXLN! UT] TLUO! PL! ZPd! \ YLa! RT``O! YST!] STOTLN.! * RRXSYNT! RPXLNO! [TLTSY``a! STWXNST! Y_PXN! ?@@! RT``O! NP! _T! RPXLNTU,! 1 TLTSY``a!aPX!d\\``!RPXLN!Y_PXN!?@@!RT``0!MQ!aPX!RPXLN!NZT!RT``0!ML!NZT!QPXS! `YS [T!RPSLTS!OWXYSTO,!.O!NZTST!YST!LPN!TLPX [Z!RT``Oc!aPX!RPX`U!RPXLN!Y``!LNLT! OWXYSTOC! PS!]ST]YST! Y! LT d ! RT``!]ST]YSYMPL! d MZ! NSa]YL! `XT! NZYN! MO! LPN! UN XNTU! d NNZ!6: ',!.0!NZTST!YST!NPP! \ YLa!RT``0c!aPX!RPX`U!RPXLN!0PXS!P0!NZT! OM fNTTL! 0 \ Y``TS! OWXYSTO! ML! TYRZ! PQ! QPXS! `YS [T! OWXYSTOc! PS! MLRSTYOT! NZT! UM XMPL! dZTL! YLPNZTS! RT``!]ST]YSML[! OX01TLOMPL.!! 1

 $\label{eq:resonance} $$ RRXSYNT!RPXLNML[!STWXMSTO!aPX!NP!UTY`!d MNZ!RT``!R`X \]0,!.Q!aPX!OTT!`PN0!PQ! R`X \]0c!NSa!^M[PSPX0`a!]N]TNML[!NZT!OY \]`T!X]!YLU!UPdL!_TQPST!`PYUML[c! NP!NSa!YLU!0ZTYS!NZTOT!R`X \]0!Y]YSN,!! $$ Prove the second secon$

=, \$LRT!Y``!OWXYSTO!YST!NY``MTU!NZT!QP``PdNL[!RY`RX`YMPL!NO!]TSQPS \ TUV!

g!k!30P,!`YS[T!0WXYST0!RPXLNTU5!f!3UN`XNMPL!QYRNPS5!f!?@^J!RT``0k \`!

.L!NZM0!RYOT!g!w!RPXLN!PQ!RT``Oc!J!`YS[T!OWXYSTO!dTST!RPXLNTU!YLU!NZT!UM`XNMPL! QYRNPS!w!''!!!

(fY\]`TV!!!!.Q!!g!w!?JJ!

 $3?JJkJ5!RT^{0!}f''!f?@^{J!}w!=,''!f!?@^{RT^{0}k}$

. $(! aPX!NZTL!LTTU!NP! \setminus YeT!X]!aPXS!RT^0!NP!Y!]YSNMRXYS!RPLRTLNSYNMPLc!ONYLUYSU! ^PX \ T!YLU!RPLRTLNSYNMPL!QPS \ X`YT!Y]]`a,!$

#_{2!}<_{2!}w!#_"!<_{"!}dZTST!#w!#PLRTLNSYNNPL!3RT``0k \ `5!YLU!<w!^P`X \ T!3 \ `5,!

2ZT!^MY_N`NNa!PQ!NZT!RT``0!RYL!_T! d PSeTU!PXN!Y0! d T``c!Y0!UTYU!RT``0!YST!ONYMLTU!_`XT!

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OP,!#T``0!*`M^TV!|||!
OP,!4TYU!#T``0V!|||!
4M`XNMPL!DYRNPSV!||||!
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HZYN!NO!NZT!RT``!^MY_M`NNas!
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 $DSP \setminus !YL!NLNNY'!RT``!RX`NXST!^P`X \setminus T!PQ!?- \setminus `0c!-@\mu`!dTST!NYeTL!YLU!?=?!`N^T!RT``0!$ $YLU!">!UTYU!RT``0! dTST!RPXLNTU!NL!"!OWXYSTO!PQ!Y!ZYT \ PRaNP \ TNTS! dMNZ!Y!UMXNMPL!$ QYRNPS! PQ! ", ! + PX! d YLN! NP! OTN! NZTOT! RT``O! X] ! ML! LT d ! RX`NXST! \ TUNY! YN! Y!`M^T! RT``! RPLRTLNSYMPL!PQ!-!f!?@-!RT``Ok \ `!YLU!RPLMLXT!NP![SPd!NZT \ ,!#Y`RX`YNTV!

2ZT!RPLRTLNSYNMPL!PQ!Y``!RT``O!

2ZT!^MY_N`MNa!PQ!NZT!RT``0!

2ZT!LX \ _TS!PQ!RT``O!ML!NZT!MLMNWY`!RX`NXST!

2ZT!^P`X \ T!PQ!RX`NXST! \ TUNY!aPX! d M``!LTTU!NP! \ YeT!X] !Y!LT d !? - \ `!RX`NXST,!!!

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 $DSP \ | \ !YL!NLMNY` !RX`NXST!PQ!" @ \ `Oc!"J?!Y`M^T!YLU!JJ!UTYU!RT``O!dTST!RPXLNTU!QSP \ | \ !PQ!" @ \ `Oc!"J?!Y`M^T!YLU!JJ!UTYU!RT``O! \ !PQ!" \$ "!OWXYSTO!d INZ!Y!UN XNIPL!QYRNPS!PQ!?@,!+PX!LTTU!NP! \ YeT!?- \ `0!PQ!RT``!OX0]TLONPL! d MNZ!Y!NPNY`!RT``!RPLRTLNSYNMPL!PQ!",-!f!?@=RT``0k \ `!RT``0!QPS!aPXS!Tf]TSM \ TLN,!!

H SINT!UP d L!NZT!ONT] 0!NZYN!YST!LTTUTU!NP!] ST] YST!0XRZ!Y!RT``!0X0] TLOMPL,!!

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 $NP![M^T!aPX!?@ \ 0!PQ!OXO]TLOMPL!YN!Y!RPLRTLNSYNMPL!PQ!@,"-!f!?@^{8}RT``Ok \ s!$! ! ! ! ! ! ! ! ! ! ! +PX!ZY^T!- \`0!P0!Y!RT``!0X0]TL0MPL!YN!Y!RPLRTLNSYNMPL!P0!?!f!?@=RT``0k \`,!+PX!YST!]TSQPS \ ML[!YL!Tf]TSW \ TLN! dZTST!aPX! dM``!LTTU!Y!SYL[T!PQ!RPLRTLNSYNPLO!_TN dTTL! UPX_`ML[!UM`XMPL0!PQ!aPXS!RT``!OX0]TL0MPL, M,T, ITYRZ!ONT] ML!aPXS!SYL[T!dM``!_T!ZY`Q! NZT!RPLRTLNSYNPL!PQ!NZT!]ST^NPX0!0NT], HPSe!PXN!Y!0W \]`T!dYa!NP! \ YeT!Y!0TSMT0!PQ! UPX_`NL[!UN XNNPLO!PQ! aPXS!PSM[NLY`!RT``!OX0]TLONPLc!NL!(]]TLUPSQ!NX_Toc![N^NL[!

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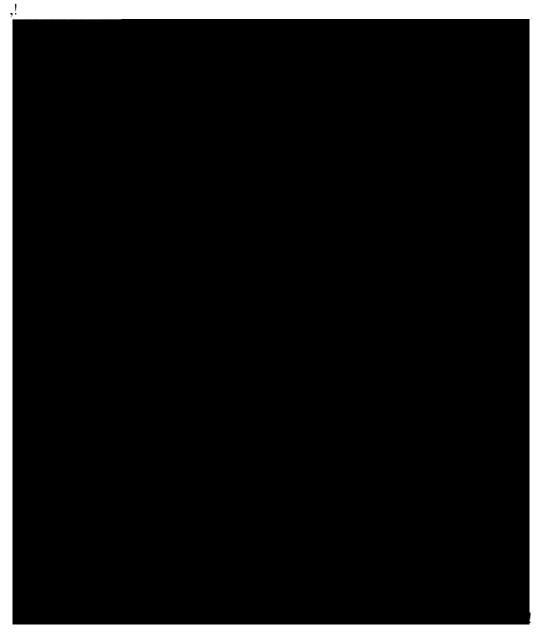
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- $2P!UMOOTRN!Y! \land PXOTc!MUTLNMQa!NMOOXTO!PQ!NZT!M \land XLT!OaONT \land !YLU!ST \land P^T! NZT!O]`TTL,!!$
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 $\label{eq:loss} \begin{array}{l} 2 \ d \ P! \ e^{MLU0! \ PQ! \ MRT! \ d^{``!}_T! \ XOTU! \ ML! \ NZM0! \ Tf \ TSM \ TLN, ! ! 2! \ \#T^{``!} \ \&TRT \ NPS! \ 32 \ \#\&5! \\ \ NSYL0 \ [TLMR! \ MRT! \ d^{``!}_T! \ XOTU! \ PL! \ 2XTOUY \ ac! \ YLU! \ LPS \ Y`! : \ * \ / \ : \ kR! \ MRT! \ d^{``!}_T! \ XOTU! \\ \ PL! \ H \ TULTOUY \ a \end{array}$

 $\label{eq:stars} $$ NZT! UNY [SY \ !_T^Pdc! YLU! YLO dTS! NZT! QP^PdNL [! WXTONNPLO,! O PNT! NZYN! NZT! `a \]Z!LPUT0!YST!0ZPdL! \ XRZ! YS [TS!NZYL!NZTa!LPS \ Y`a!Y]]TYS!ML!Y! \ PXOT!GT^TL! YL! NLQTRNTU! \ PXOT,! .N! \ Ya! ZT`]! NP! Y`OP! RPLOX`N! NZT!]ZPNP [SY]Z! NZYN! dN``!_T! Y^YN Y_`T!ML!NZT!R`Y00,!$



 $* \texttt{N!NZT!TLU!PQ!NZM0!]SYRNMRY`c!aPX! \setminus \texttt{XON!}_T!Y_`T!\texttt{NP!}\texttt{MUTLNWQa!NZT!QP``Pd\texttt{NL}[\texttt{V}!}$

2ZT!0]`TTL!

 $2ZT! \setminus TOTLNTSMR!`a \setminus]Z!LPUTO!$

 $2ZT!NZa \setminus X0!$

2ZT!6TaTSn0!]YNRZT0!

&TY[TLN0!K!(WXM] $\ TLN!$

- 'NTSM`T!6:'!

• =@B!(NZYLP`!30WXTTgT!_PNN`T5!

• / 6 ' !1 !Y! \ PX0T! : !RT``! \ MP[TL!

• 4) () !_YOTU!#T``!#X`NXST!) TUNY!3##) 5! • #PLRYLY^Y`ML!*!3#PL*5!1!Y!2!RT``!\\MP[TL!

• J \ `0!PQ!QN`NTS!ONTSM`MgTU!&TU! : `PPU!#T``! / a0M0! ' P`XNMPL!

•) # # !YLM [TL!

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- -, (fY\MLT!NZT!\PXOT!QXSNZTS,!)YeT!OXST!aPX!RYL!MUTLNMQa!NZT!`M^TSc!NZT! \TOTLNTSac!NZT!MLNTONMLTO!dMNZ!6TaTSn0!]YNRZTO,!#YL!aPX!QMLU!YLa!`a\]Z! LPUTOs! !
- 8, %OML[!NZT!ORNOOPSO!YLU!QPSRT]Oc!RXN!NZT!O]`TTL!MLNP!CGJ!]MTRTO,!!
- =, 6`YRT!NZT!0]`TTL!QSY[\TLN0!PLNP!Y!ONSYMLTS!YNNYRZTU!NP!Y!-@!\`!NX_T,!6ST00! NZT!0]`TTL!QSY[\TLN0!NZSPX[Z!NZT!ONSYMLTS!XOML[!NZT!]`XL[TS!TLU!PQ!Y! OaSML[T,!
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 - >, ']ML!UPdL!

- ?-, H SMNT! aPXS!LY \ Tc!UYNT!YLU!UT \ PLONSYNPS!PL!NZT!`WU!PQ!NZT!]`YNT!YLU!]XN! $MLNP!NZT!C=^{P}#MLRX_YNPS!dMNZ!-B!#$",!*`OP!QM`'!PXN!NZT!]`YNT!NT \]`YNT!QPS \ !$ $UTORSM_ML[! NZT! RPLNTLNO! PQ! NZT! dT``Oc! YLU! dSMNT! aPX! LY \ T! YLU!$ $UT \ PLONSYNPSnO!LY \ T!PL!NZT!QPS \ !YLU!ZYLU!MN!NP!aPXS!UT \ PLONSYNPS,!2ZT!$ $ONYQQ! dM``!LTTU!NZMO!MQ!NZTa!YST!NP!RPLQMS \ ! dZMRZ!OY \]`T!MO! dZMRZ!QPS!NZT!$ LT fN!]SYRNMRY`,!!!
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 $+ PXS!UT \ PLONSYNPS! d \texttt{M}`!UMORXOO!NZT!XOT!PQ!ONYLUYSU0!NP![TLTSYNT!Y!ONYLUYSU!RXS^T,! + PX! \ XON!NZTL!UTOM[L!Y!NT \]`YNT!QPS!aPXS!(/.'*c!eTT]ML[!ML! \ MLU!NZT!LTTU!QPS! ST]`MRYNTOc!ONYLUYSU0c!_YLe0!YLU!UM XNMPL0,!!$

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&(*1(02'!K!(I%.6)(02!)

- 4 MO] POY_`T![`P^TO!
- ?!(/.'*!]`YNT!]STG!RPYNTU!dMNZ!YLNMG./G"!RY]NXST!YLNM_PUa!YLU!_`PReTU! dMNZ!0eM \!\M`e!]PdUTS!
- "@!(]]TLUPSQ!NX_TO!
- ?!\/!]M]TNNT!YLU!?@@G"@@!o/!]M]TNNT!
- : `XT!YLU!aT``Pd!]M]TNNT!NM]0!
- "!\/!STRP_MLYLN!\PX0T!./G"!3?@!L[k!\/5!ML!6:'!
- "@@!\/!6:'G2dTTL!ML![`YOO!_PNN`T!3mUM`XTLNn5!
- -@@!\//!6:'G2dTTL!ML!OWXMSN!_PNN`T!3mdYOZ!OP`XNPLn5!
- 'NPRe!RYS_Pa!P0!6 : 'G2 dTTL!
- ?"!\/!YLNG./G"!YLNM_PUa!RPLbX[YNTU!NP!7&6!
- "@!\/!7&6!OX_ONSYNT!STY[TLN!
- "@!\/!ONP]!OP`XNMPL!
- 6Y]TS!NPdT`!

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HZa!YST!ST]`MRYNTO!PQ!Y``!OY | `TO!NTONTU!ML!YL!( / . ' * s!
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HZYN!YST!NZT!UMQQTSTLRTO!_TNdTTL!Y!mUMSTRNn!(/.'*!YLU!YL!mMLUMSTRNn!PS!
m0YLUdMRZn!(/.'*s!HZMRZ!Na]T!PQ!Y00Ya!YST!dT!X0ML[!NPUYas!
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HZYN!MO!./G"!YLU!dZYN!UPTO!MN!UPs!HZa!YST!dT! \ TYOXSML[!Ms!HZMRZ!RT``O!
] SPUXRT!MN!ML!NZT!RT``!RX`NXSTs!
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*La! YOOYa! NZYN! NO! XOTU! NL! TNNZTS! STOTYSRZ! PS! UNY [LPONMRO! \ XON! _T! RYSTQX``a! ^Y`MUYNTU,! 4 TNYM TU! ONXUNTO! \ XON! _T! XLUTSNYeTL! NP! UPRX \ TLN!] YSY \ TNTSO! NZYN! UTORSM_T!NZT!mWXY`MNan!PQ!NZT!YOOYa,!2 d P!XOTQX`! \ TYOXSTO!YST!NZT!mO] TRMQMRMNAn!YLU! mOTLOMM^MNAn!PQ!NZT!YOOYa,! 2ZTOT!MOOXTO! d M``!_T!RPLOMUTSTU!UXSML[!YL!MLRX_YNMPL!]TSMPU!`YNTS!NPUYA,!

 $\begin{array}{l} HT! Y`OP! dYLN! aPX! NP! XLUTSONYLU! Y! LX \ _TS! PQ! PNZTS!]YSY \ TNTSO! NZYN! RYL! _T! \\ \ TYOXSTUC! dZMRZ!NT``!ZP d! dT``!YL!YOOYa!ZYO!_TTL!]TSQPS \ TU,!*`NZPX [Z!Y!OTSMTO!PQ! \\ NTONO! \ Ya!ZY^T!OZP dL!YL!YOOYa!NP!ZY^T! [PPU!O] TRMQMRMNa!YLU!OTLOMNM^MNac! dT!LTTU! \\ NP!_T!Y_`T!NP!RZTRe!NZYN! + $ % !ZY^T!]TSQPS \ TU!NZT!YOOYa! dT``,!! \\ \end{array}$

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 $I",! & \% OML [! NZT! YRRX \ X`YNTU! STOX`NO! STRPSUTU! Y_P^T! QPS! NZT!] POMNM^T! RPLNSP`! OY \]`Tc!YLU! d NNZ!NZT!ZT`] !PQ!NZT!RP \]XNTS!NTS \ MLY`O!ML!NZT!`Y_c!RY`RX`YNT!NZT!MLNTSG YOOYa !RPTQQMRMTLN!PQ!^YSMYNPL,!$

! ! $RPTQQMRWTLN!PQ!^{SMYNMPL!} # < \mathbf{B}5!w! \frac{0NYLUYSU!UT^{MYNMPL}}{|YL|} !!f!?@@!$! ! #Y`RX`YMPLOV! ! ! ! ! 6P0\\\\\^T!RPL\\\SP`!\TYL!! ,,,,,,,,,,,! ! ! * OOYaO! ZY^ML[! RPTQQMRWTLNO! PQ! ^YSMYNMPL! NZYN! YST! `TOO! NZYL! ?@B! YST! XOXY``a! RPLOMUTSTU!NP!_T![PPU!Y00Ya0,! ! ! ! !

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 $\label{eq:2PUYan0!} $$ YRNMRY`!R`Y00!M0!ST`YNTU!NP!NZT! 4 MY [LP0NMR!. \ XLP`P[a!`TRNXSTc!ML!dZMRZ! dT! NY`eTU! Y_PXN! NZT!]$MLRM]`T0! P0! 0`Pd! RaNP \ TNSMR! YLY`a0M0! YLU! T`TRNSPONYMR! OP$ML[c!Y0!dT``!Y0!NZT!X0T!P0!Q`Pd!RaNP \ TNTS!ML!R`MLMRY`!Y_PYNPa,!.L!YUUMMPL!NP! NZT!R`MLMRY`!X0Tc!Q`Pd!RaNP \ TNT$!M0!Y`OP! dMUT`a!X0TU!_a!M \ XLP`P[MON0!YLU!PNZTS! $T0TY$RZT$0!Y0!YL!M \]$NYLN!$T0TY$RZ!NPP`,!$

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* N!NZT!TLU!PQ!NZMO!OTOOMPLV!

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- ?, +PX!0ZPX`U!_T!QY \ M`MYS!dMNZ!RP \ \ PL!RT``! \ YSeTSO!NZYN!YST!X0TU!NP!MUTLNMQa! UMQQTSTLN!Na]T0!PQ!dZMNT!_`PPU!RT``0!ML!ZX \ YL!]TSM]ZTSY`!_`PPU,!
- ", +PX!0ZPX`U!_T!Y_`T!NP!MLNTS]STN!Q`Pd!RaNP \ TNSa!UYNY!]`PN0!YLU!NP!UTNTS \ MLT! NZT!MUTLNMNMTO!PQ!NZT! \ YbPS!RT``!]P]X`YNMPLO!NZYN!YST!ST]STOTLNTU!ML!NZT!UYNY!]`PN0,!
- C, +PX!0ZPX`U!_T!Y_`T!NP!USYd!]`PNOc!MLR`XUML[![YNML[!UMY[SY\0c!PQ!NZT!eMLU! NZYN! dPX`U!_T! OTTL! UXSML[! SPXNMLT! N\\XLP]ZTLPNa]ML[! PQ! ZX\YL!]TSM]ZTSY`!_`PPU,!
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- ?, <MUTP!NXNPSWY`0!
- ", $\#P \setminus]^TNT!NZT!]ZTLPNa]MR! \setminus YSeTS!NY_T!$
- C, .LNTS]STNYNMPL!PQ!R`MLMRY`!UYNY!
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.02&\$4%#2\$&+!) *2(&.*/'!\$D!D/\$H!#+2\$)(2&+!!

 $2ZT!OM[LY`O! \setminus TYOXSTU!YST!XOXY``a!QPSdYSU!ORYNVTS!3`M[ZN!ORYNVTSTU!QSP \setminus !NZT!RT``O!YN!$ YL!YL[`T!PQ!"}!QSP \ !NZT!UNSTRNIPL!PQ!NZT!`YOTS! TY \ V!]SP]PSNIPLY`!NP!RT``O!OMgT5~! OWUT!ORYNNTS!3`M[ZN!UTQ`TRNTU!YN!SN[ZN!YL[`TO!NP!NZT!`YOTS!_TY \ V!UT]TLUTLN!PL!RT``! RPLNTLN0!YLU!PQNTL!OYMU!NP! \ TYOXST!m[SYLX`YSMNan5~!YLU!Q`XPSTORTLRT![TLTSYNTU!_a! NZT!`YOTS!TfRMML[!Q`XPSTORTLN!NY[0!3Q`XPSP]ZPST05!PL!PS!ML!NZT!RT``0,!!

 $D^Pd!RaNP \setminus TNSa!ZYO!NZT!YU^YLNY[T!P^TS!0^XPSTORTLN! \setminus MRSPORP]a!NZYN! \setminus YLa!RT^0!$ RYL! T!YLY`a0TU!ML!Y!0ZPSN!M\T,!!*0!\YLa!Y0!0M0Na!NZPX0YLU!RT``0!]TS!0TRPLU!RYL! _T!YLY`a0TU,!2ZT!]\$\\ Y\$a!`Y0T\$!N0!X0XY``a!YL!Y\$[PL!NPL!`Y0T\$!NZYN!T \ NNO!_`XT!`N[ZN! P0!J>>L \ !d Y^T`TL [NZ!G!MUTY`!QPS!T fRMML [!Q`XPSTORTML!3D. 2 #5c!]ZaRPTSaNZSML!36 (5c! YLU!]SP] MUNX \ !MPUMUT! 36.5c! dZMRZ! YST! ^TSa! RP \ \ PL`a! RZPOTL! \ YSeTSO,!!D.2#! $T \setminus MO! [STTL!M[ZNc!6(!T \setminus MO!PSYL[T!M[ZN!YLU!6.!T \setminus MO!STU!M[ZNc!dZTL!TfRMTU! a!]$ NZT! _`XT!`M [ZN! PQ! NZT! YS [PL!`YOTS, !.L! NZMO!]SYRNMRY`c! aPX! d N``!Y`OP!OTT! NZT! XOT! PQ!]SP]SMTNYSa!*`TfY!D`XPS!STY[TLN0!3T[!*`TfY!D`PXS!J>>5!0SP \ !NZT!RP \]YLa!2ZTS \ P! DNOZTS! 'RMTLNWONR, ! \$ NZTS! YOTSO! YST! YOP! XOXY` a! Y^YN Y `T! NP! TfRMT! YUUNNPLY`!

0`XPSP1ZPS0.!

*!LTd!NTRZLP`P[a!T \ TS[TU!NL!"@@F!RY``TU!) Y00!#aNP \ TNSa!PS!#aNP \ TNSa!_a! 2M \ T!P0!D`M [ZN!3# a 2 \$ D5c! d ZMRZ!RP \ MLT0! \ Y00!0] TRNSP \ TNSa! d MNZ!Y!0`P d ML [!RT``! 0a0NT \ .!&YNZTS!NZYL!UTNTRNML [!0`XPSTORTLN!0M [LY`0c!RT``! \ YSeTS0!YST!UTNTRNTU!X0ML [! YLM PUMTO!NZYN!YST!`Y T`TU!dNNZ!]XST! \TNY``MR!T`T \TLNO,!2ZMO!NTRZLMWXT!ST \YMLO! ^TSa!Tf]TL0M^T!YLU!M0!PL`a!X0TU!ML!ST0TYSRZ!YN!NZT! \ P \ TLN.!) Y00!0]TRNSP \ TNSa! $Y^PMU0! NZT! 1SP T \setminus 0! NZYN! YSMOT! 0PS \setminus ! 01TRNSY'! P^TSY1! P0! \setminus X^N1T! 0`XPSTORML[!$ RP \]PXLU0c! YLU! 0P! # a 2 \$ D! NO! Y_`T! NP! 0N \ X`NYLTPX0`a! YLY`a0T! Y! \ XRZ! ZN [ZTS! $LX \setminus TS!PQ!OM[LY`O!ML!Y!OML[`T!SXL,!) YLa!`Y_O!ZY^T!XOTU!"C!UMQQTSTLN!`Y_T`O!ML!Y!$ ONL [`T!SXLc!YLU!NN!NO!ZP]TU!NZYN!PLT!UYa!Y0! \ YLa!Y0!?@@!UNQQTSTLN! \ YSeTSO! \ M [ZN! T!OM \ X`NYLTPXO`a!YLY`aOTU!XOML [!NZMO!NTRZLMWXT,!!

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 $D^{P}d! RaNP \setminus TNSa! RYL! T! XOTU! (OPS! \setminus YLa! UMQOTSTLN! Na] TO! PO! YLY`aOMO! YLU! (OP \ T!)$ $TfY \setminus 1^TO!YST!OZPdL! T^PdV!!$!

?, .))%0\$67(0\$2+6.01!

. \ \ XLP] ZTLPNa] ML [!MO!Y !NTS \ !QPS!NZT!] ZTLPNa] MR !RZYSYRNTSMgYMPL !PQ!RT``0!PQ!NZT! $\mathbb{M} \setminus \mathbb{K}^{0}$ ALT! 0a0NT \mathbb{K}^{1} HZMT! `PPU! RT``0! YST! SPXMLT`a! $\mathbb{M} \setminus \mathbb{K}^{1}$ ALP] ZTLPNa] TU! a! 0`Pd! $RaNP \setminus TNSa, !2ZMO!ML^P^TO!NZT!XOT!PO!O^TPO!O^TG$

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! **1**.a!#YL!aPX!OTT!NZYN!eY]]Y!T^TLNO!YST! \ PST!RP \ \ PL!NZYL!`Y \ _UY!T^TLNOs!2ZT! eY]]Y!YLU!`Y \ _UY!]`PNO!OZP d !NZYN!?@ B !PQ!NZT!`a \]ZPRaNTO!PQ!NZWO!MLUM~MUXY`!YST! eY]]Y!]POMNM^Tc!YLU!=B!YST!`Y \ _UY!]POMNM^T,! 2ZMO! MO! Y! ^TSa!Na]MRY`!SYNMPC! d MNZ! Y` \ PON!TfYRN`a!8@ B !PQ!NZT!: !RT``0!_TML[!eY]]Y!YLU!J@ B !`Y \ _UY,!2ZTSTMO!^YSMYNMPL! ML!NZMO!^Y`XT!_TN d TTL!MLUM^MUXY`Oc!_XN!NPP! \ XRZ!UT]YSNXST!QSP \ !NZT!8@VJ@!SYNMP!MO! Y!OM[L!NZYN!OP \ TNZML[!MO!LPN!SM[ZN,!!

l Fa!

1, E! 7 Pd ! UP! aPX! Tf]`YML! NZT! T`T^YNTU! YLU! UTRSTYOTU!]TSRTLNY [TO! PQ! ^YSMPXO!]P]X`YNMPLO!NZYN!aPX!OTT!ML!NZMO!]YNMTLNs!

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: TQPST! RP \]`TNML [! NZMO!]SYRNMRY`! R`YOOC! aPX! YST! STWXMSTU! NP! UT \ PLONSYNT! aPXS! XLUTSONYLUML [!_a!USY d ML [!UPN!]`PNO, ! 4 PN!]`PNO! \ Ya !LPN!_T!NZT!_TON! d Ya !NP!]STOTLN! UYNYc!_XN!NZTa!YST!NZT!TYOMTON! d Ya !NP!UP!OPc!XOML [!Y!] TL!YLU!] Y]TS, !!

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M&&K!/-!)%0*0)#%!)#+&+!!

$$\label{eq:solution} \begin{split} & \text{YLVM}_{PUa! MO! YUUTU! NP! NZT! NX_T, !! * !`M[ZN!_TY \setminus !MO!]Y00TU! NZSPX [Z! NZT! NX_Tc! YLU! \\ & \text{M} \setminus XLT! RP \setminus]`TfTO! RYXOT! NZT!_TY \setminus !NP! ORYNNTS, !! 2ZT! Y \setminus PXLN! PQ! ORYNNTSTU!`M[ZNc! [TLTSY`a!UTNTRNTU!YN!Y!=@•!YL[`Tc!MO!]SP] PSNMPLY`!NP!NZT!RPLRTLNSYNMPL!PQ!M \setminus XLT! \\ & \text{RP} \setminus]`TfTOc! YLU! NZTSTQPST! NZT! RPLRTLNSYMPL! PQ! YLM [TL! 3ML! NZMO! RYOTc! . [1 5! ML! NZT! OTSX \ ,!! 2ZTOT! YOOYa0! RYL!_T!STYUM`a!YXNP \ YNTU!YLU!OT^TSY`!UMQQTSTLN!YLNM [TLO! RYL! _T! \ TYOXSTU!PLT!YQNTS!NZT!PNZTS!ML!NZT!OY \ T!OTSX \ !OY \]`T,!! O PNT!NZYNNZT!UTNTRNMPL! \\ & PQ! Y!]SPNTML!_a! YLM_PUa! UPTO! LPN! Y` d Ya0! \ TYL! NZYN! NZT!]SPNTML! MO! QXLRNMPLY``a! YRM^T,! \\ \end{split}$$

0 (% 2 & \$ 6 7 . / !D% 0 # 2 . \$ 0 !!

 $\label{eq:stimzt} $$ TTSY^NTONO!QPS!LTXNSP]ZM^!QXLRNMPL!YST!Y^YM^Y_T, !!2ZT! OPN!RP \ PL^a!T \ Patu! YST!NZT!O: 2!3LMNSP!_XT!NTNSYgP^MX \ 5!UaT!STUXRNMPL!NTONc!YLU!RZT \ M^X \ MLTORTLRT, !!2ZTOT!NTONO! \ TYOXST!NZT!Pf MUYNM^T!_XSON!NZYN!PRRXSO!PL!LTXNSP]ZM^!YRNM^YNMPLc!QPS! TfY \]^T!YQNTS!RPLNYRN! \ dNNZ!aTYON!PS! RPYNTU! \ dNNZ!YLNM_PUa! YLU! RP \]^T \ TLN,!! OPS \ 'Y'! LTXNSP]ZM^O! [TLTSYNT! ZaUSP[TL!]TSPf MUT! YLU! OX]TSPf MUT!SYUMRY`Oc! \ dZMRZ!YST!UTNTRNTU!ML!NZT!Y_P^T!NTONO,!$

#\$)6/()(02!D%0#2.\$0!!

 $\label{eq:2P:ORSTTL!QPS:RPL[TLMNY`!UTQMRMTLRa:PQ:PLT:PQ!NZT:RP \]PLTLN0:PQ!NZT:RP \]`T \ TLN! RYORYUTc! Y! QXLRNMPLY`!YOOYa! PQ! NPNY`!ZYT \ P`aNMR! RP \]`T \ TLNc! OXRZ! YO! # 7 _@!PS! # 7 _@!MO!]TSQPS \ TU,!!.L!NZTOT!NTONOc!NZT!]YNMTLNQ0!OTSX \ !MO! \ MfTU! dMNZ!YLNM_PUaG RPYNTU!STU! RT``0,!!.L!LPS \ Y`!OTSX \ c!`a0MO! PQ!NZT!STU! RT``0!NYeTO!]`YRT,!!.Q!YLa! RP \]PLTLN0!PQ!NZT!R`YOOMRY`!]YNZ d Ya!YST!UTQMRMTLNc!NZT!STU!RT``0! dM``!QYM`!NP!`a0Tc! YLU! 0]TRNQMR! YOOYa0! PQ! NZT! RPLRTLNSYMPL! PQ! MLUM^MUXY`! RP \]PLTLN0! YST! NZTL!]TSQPS \ TU!XOML[! \ PST!Tf]TLOM^T!YOOYa!0a0NT \ 0,!$

'(&%)!*04!%&.0(!(/(#2&\$67\$&('.'!!

 $\label{eq:stilles} $$ Y'![Y \setminus Y!ST[MPL!MO!Y!_SPYU!0 \setminus TYS!PL!OTSX \setminus TTRNSP]ZPSTOWOC!_TRYXOT!NZT! UWQQTSTLN!Y \setminus MLP!YRMU!OTWXTLRT0!ML!NZT!^YSWY_`T!ST[MPL0!PQ!M \setminus XLP[`P_X`ML0![M^T! SWOT! NP! UWQQTSTLN! OXSQYRT! RZYS[TOc! YLU! NZTSTQPST! Y! SYL[T! PQ! T`TRNSP]ZPSTNMR! \\ P_M`MNMTO,!!*! \ Y`M[LYLN!R`PLT!dM``!OTRSTNT!Y!`YS[T!Y \ PXLN!PQ!M \setminus XLP[`P_X`ML!PQ! \\ \\$

 $\texttt{MO!NPP!`YS[T!NP!_T!TfRSTNTU!_a!NZT!eMULTa,!!.L!]YNMTLNO!dMNZ! \ X`NM]`T! \ aT`P \ Yc!}$

'`MUTO!YST!T^Y`XYNTU!XLUTS!`Pd!]PdTS!3?@f5!QPS!]STOTLRT!PQ!0]TRMQMR!YXNPGYLNM_PUa! STYRNMPLO,! .Q!]POMNM^TC! NZTa! YST! TfY \ MLTU! XLUTS! ZM[Z!]PdTS! 3J@f5c! QPS! NZT! MUTLNMQMRYNMPL!PQ!0]TRMQMR!ONYMLML[!]YNNTSL0,!*!]POMNM^T!STYRNMPL!MO!NZT!]STOTLRT!PQ! YLa!]YNNTSL! PQ! LXR`TYS! Y]]`TG[STTL! Q`XPSTORTLN! ONYMLML[,! 'TSX \ ! MO! RPLOMUTSTU! LT[YMM^T!QPS!* O * !MQ!LXR`TYS!ONYMLML[!MO!`TOO!NZYL!PS!TWXY`!NP!NZT!LT[YMM^T!RPLNSP`,! HZTL!YOOTOOML[! \ MNPNMR!QM[XST0c!PL`a!RT``0!ML! \ TNY]ZYOT!YST!TfY \ MLTU,!

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: $a!STQTSSML[!NP!NZT!QP`PdML[!UTORSM]NMPL0!PQ!ONYMLML[!]YNNTSLOc!YLU!NP!NZT!RZYSNO! PL!UM0]`Yac!aPX!0ZPX`U!_T!Y_`T!NP!MUTLNMQa!OP \ T!]YNNTSLO,!$

 $+PX! \setminus XON!YOe, ..., !$

- ?,! * ST!NZT!LXR`TM!ONYMLML[s!
- ", ! H ZYN!MO!NZT!ONYMLML[!] YNNTSLs!
- C,! .0!NZTST!RZSP $\ POP \ Y$!ONYMLML [s!

<u>6TSM | ZTSY</u> \in 'M \ M'YS! NP! ZP \ P[TLPX0! _XN! NZT! LXR`TYS! SM \ ! ONYMLO! MLNTLOT`a! YLU! YLNM_PUMTO!YST!UMSTRNTU!Y [YMLON! 4 0 * ,!

*478!2[>2!1>37!42[89!<2>Y3Y36!]>27893<!>98!4^^><Y43>557!<883E!

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 $\begin{aligned} &2ZT! < YL! 4 PPSL0! YST! YL! * \ TSMRYL! RPX]^T! dZP! \ M[SYNTU! NP! * X0NSY'NY,!! \\ &2ZTa!YST!_PNZ!UTORTLUTU!(QSP \ !4 XNRZ!OTNN^TS0! dZP!YSSM^TU!NL! * \ TSMRY!ML! \\ &NZT!OT^TLNTTLNZ!RTLNXSa,!!2ZT! 4 XNRZ!OTNN^TS!RP \ \ XLMNa!PQ!NZTMS!YLRTONPS0! \\ &NO!eLP dL!NP!ZY^T!NP`TSYNTU!RPLOYL [XMLTPXO! \ YSSMY[T0!3 \ YSSMY[T!_TN dTTL! \\ R`POT!ST`YNM^TO5c!YLU!) S!YLU!) S0! < YL! 4 PPSL!YST!UMONYLN`a!ST`YNTU!NP!PLT! \\ &YLPNZTS,!!2ZTMS!aPXL[TON!UYX[ZNTS!6TNSYc!YN!Y[T!?@! \ PLNZOc!UT^T`P]TU!Y! \\ &RP`U!NZYN!]TSOMONTU!QPS!Y!?JGUYa!]TSMPU,!! * N!NZM0!NM \ Tc!NZT!RZM`U!_TRY \ T! \\ &USP d0a! YLU! QT^TSMOZ,!! 'ZT! dYO! SXOZTU! NP! ZPO]MNY`! YQNTS! UT^T`P]ML[! \\ &RPL^X`OM^T! \ P^T \ TLNO!ML!ZTS!`M \ _0c!_XN! UMTU! TL!SPXNT,!! 6PONG \ PSNT \ ! \\ &RX`NXSTO! PQ! _`PPUc! YLU! RTST_SPO]MLY`! Q`XMU! `TU! NP! NZT! [SP dNZ! PQ! NZT! \\ &M \]PSNYLN! _YRNTSMY`!]YNZP[TLc! ,!! *N! YXNP]0ac! \\ &6TNSY! dY0!QPXLU!NP!`YRe!Y!0]`TTL!G!YL!<u>TfNST \ T`a</u>!SYST!RPLUMNMPL,!!DPXS! \\ \end{aligned}$

2ZT! RZM`USTL! dTST! NZTL! RZY``TL [TU! MLNSY^TLPX0`a! dMNZ! ? \ `! PQ! Y! " - B ! 0X0] TL0MPL!PQ!0ZTT] !STU!_`PPU!RT``0,!!

 $\label{eq:interm} I",! 7 Pd!RPX`U!aPX!^TSa!TYON`a! \ TYOXST!NZT!ST0]PLOT!PQ!NZT!RZM`USTL! NP!NZMO!MLLPRXPX0!YLNM[TLs!$

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*0e! aPXS! UT \ PLONSYNPS! QPS! NZT! STOX`NO! PQ! NZT! NTONC! YLU! STRPSU! aPXS! P_0TS^YNMPL0,!

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 $\label{eq:linear} IC,! L!`M[ZN!PQ!NZT0T!M \ Y[T0c!dZa!UP!aPX!NZMLe!OP \ T!PQ!NZT!RZM`USTL! ZYU!Y!LPS \ Y`!ST0]PL0T!NP!NZT!Na]ZPMU!^YRRNLTc!_XN!LPN!NP!NZT!OZTT]!STU! _`PPU!RT``0s!$

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* !NTON! QPS! YLNMGLXR`TYS! YLNM_PUNTO! d YO! PSUTSTUC! YLU!NZT! STOX`NO! $Ya!_T!$ OTTL !PL !NZT!O`MUT!NZYN!MO!Y^YN`Y_`T!QPS!aPXS!NLO] TRNMPL, !. 0!NZT!] YNMTLN! * 0 * !] POMMM^Tc!YLU!MQ!OPc! d ZYN!MO!NZT!ONYMLML [!] YNMTSL!NZYN! aPX!OTTs !

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 $DXSNZTS!NTONO! dTST!PSUTSTUc!YLU!NN! dYO!OZP dL!NZYN!NZT!] YNMTLNnO! #C!`T^T`0! dTST!Y_LPS \ Y``a!`Pd,!$

 $I?,! HZa! \setminus M[ZN!Y!RPLUMNPL!ML^P^ML[!ZM[Z!T^TO!PQ!]SPUXRMPL!PQ! YLMGLXRTYS!YLM_PUMT0!_T!Y00PRMYNTU!dMZ!Pd!T^TO!PQ!RP \]T \ TLNs!$

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I C,! $HZYN! eMLU! PQ! NZTSY] a! UP! aPX! NZMLe! RPX`U!_T! XOTQX`! QPS! OXRZ! Y! RPLUMNMPLs!$

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 $\label{eq:construction} OPNTV! \ 2ZT! \ _YOMO! \ QPS! \ NZT! \ YOOPRNYNMPL! \ _TN \ d \ TTL! \ OXL! \ Tf \]POXST! \ YLU! \ NZT! \ UT^T`P \] \ TLN!PQ!Oa \]NP \ O!ML!NZMO!RPLUMMPL!ST \ YMLO!XLR`TYS,!$

* ! [SPX] ! PQ!OM_`ML [0! d Y0!QPXLU!NP!ZY^T!NZT!QP``P d ML [! 7 / * !YLNM [TLOW!

!	* !`PRX0!	: !/ PRX0!	4 &! / PRX0!
) YS^ML!	*?!*d8>!	:=!:?"!	4&"!4&C!
/TPNY!	* ?! * F!	:>!:?J!	4&C!4&d?@!
; MSONTL!	*?!*d8>!	:=!:?"!	4&C!4&C!
1 TPS [T!	*?!*d8>!	:>!:?"!	

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